PD – 087

I Year M.Sc. (DCC) Degree Examination, January 2018
(Fresh and Repeaters) (Y2K13 Scheme)
MATHEMATICS
M – 104 : Differential Equations

Time : 3 Hours | Max. Marks : 80

Instructions : *i) Answer **any five full** questions choosing atleast **two** from **each** Part.*
*ii) **All** questions carry **equal** marks.*

PART – A

1. a) Establish the Liouville's formula for $L_nY = 0$ on I. Also discuss any one consequences of the formula.
   b) State and prove Sturm's separation theorem on the zeros of the solutions of a self-adjoint differential equations. **(8+8)**

2. a) State the existence and uniqueness theorem and hence test it for
   $\frac{dy}{dx} = y^2$, $y\ (1) = -1$, $|x - 1| \leq a$, $|y + 1| \leq b$ where a and b are constants.
   b) Solve : $y'' + \lambda y = x$, $y\ (0) = 0 = y\ (1)$ by constructing its Green's function. **(8+8)**

3. a) Obtain the general solution of the Gauss-hypergeometric differential equation about x = 0 and x = 1.
   b) Prove the orthogonal property of Chebyshev polynomials. **(8+8)**

4. a) Find the fundamental matrix and determine e^{At} of $\frac{d\underset{\sim}{x}}{dt} = A\underset{\sim}{x}$ where

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix}; \underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

P.T.O.

b) Construct Liapunov function and hence find the stability of the critical point (0, 0) of : **(8+8)**

i) $\frac{dx}{dt} = -x + 2x^2 + y^2$

$\frac{dy}{dt} = -y + xy$

ii) $\frac{dx}{dt} = x^3 - 3xy^4$

$\frac{dy}{dt} = x^2y - 2y^3 - y^5$.

PART – B

5. a) Solve the following by the method of characteristics :

i) $u_x + u_y + u = 1$ with $u = \sin x$ on $y = x + x^2$.

ii) $uu_x + u_y = 1$ with $u = 0$ on $y^2 = 2x$.

b) Solve the problem $p^2x + qy - u = 0$ with $u = -x$ on $y = 1$. **(8+8)**

6. a) Solve by the Monge's method $(1 + q)^2 r - 2(1 + p + q + pq)s + (1 + p)^2 t = 0$.

b) Classify the problem $\sin^2 x\, u_{xx} + \sin 2x\, u_{xy} + \cos^2 x\, u_{yy} = x$ and hence reduce it to its canonical form. **(8+8)**

7. a) Show that variables separable solution of the Laplace equation in cylindrical polar coordinates yields a Bessel differential equation.

b) Using the Fourier transform, solve $U_t = k\, U_{xx},\ 0 \le x < \infty; t \ge 0$ with

$U(x, 0) = f(x),\quad 0 \le x < \infty;$

$U_x(0, t) = 0,\quad t > 0.$ **(8+8)**

8. a) Find the Green's function for $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + f(x)\,\delta(t),\ -\infty < x < \infty,\ t > 0$ subject to, $u(x, 0) = 0,\ -\infty < x < \infty$.

b) Determine the solution of $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + x^2,\ 0 \le x \le 1,\ t \ge 0$ with

$$\left.\begin{aligned} u(x, 0) &= x \\ \frac{\partial u}{\partial t}(x, 0) &= 0 \end{aligned}\right\};\ 0 \le x \le 1$$

$$\left.\begin{aligned} u(0, t) &= 1 \\ u(1, t) &= 0 \end{aligned}\right\};\ t \ge 0.$$

(8+8)